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Improving the Performance of Polar Decoders Using Virtual Erasure Channels.

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Abstract — In this paper we propose the use of virtual erasure channels (VECs) to improve the performance of successive cancelation decoding algorithm used to decode polar codes. VECs are concatenated to the continuous channels such as additive white Gaussian noise (AWGN) or Rayleigh channels. The outputs of the continuous channels are passed through VECs before decoding operation. The inputs of the VECs falling into a threshold interval are erased. We also provided techniques to determine the threshold levels of VECs. For the computer simulations binary phase shift keying (BPSK) is used. It is seen from computer simulations that the proposed method significantly outperforms the performance of successive cancelation polar decoders.

***Index Terms*— Polar codes, successive cancelation, threshold levels, AWGN channel, Rayleigh fading channel.**

I. INTRODUCTION

Polar codes invented by Arikan [1] is the primary class of error correcting codes whose performances can be proven mathematically. By transmitting free bits, called information bits, over high capacity channels, and by transmitting fixed bits, called frozen bits, over the low capacity channels, polar codes can achieve the symmetric capacity under successive cancellation (SC) decoding with both encoding and decoding complexities on the order of $O(N \log N)$ where N is the frame length. Because subchannels are not completely polarized for finite length of polar codes, the error-correcting performance of successive cancellation (SC) decoding algorithm is poor at short and moderate block lengths. Although much work has been done in the area of polar decoding in recent years [2–6], it is still an open issue to find a decoding algorithm with both good frame error rate (FER) performance and low complexity, especially with finite-length polar codes.

In this paper, we propose virtual erasure channels through which received symbols are passed before they are sent to the polar decoders employing successive cancelation decoding algorithm. The input symbols of the VECs falling into a threshold interval are erased and these inputs symbols contain unreliable information about the transmitted polar code-bit. The proposed method is employed for additive white Gaussian noise (AWGN) and Rayleigh fading channels. We proposed techniques for the determination of threshold intervals. One of them is based on the values of likelihood ratios (LRs), whereas the second one considers the use log logarithmic likelihood ratios (LLRs). It is also shown in the paper that more than one VEC can be employed in parallel at the output of the wireless channels, and for such a structure we proposed a parallel concatenated decoder structure and proposed criteria for the selection of the decoders' output streams. It is shown via computer simulations that the proposed techniques achieve a significant performance improvement compared to the classical SC decoding. The outline of the paper is as follows. In Section-2, brief background information about polar codes is provided. The proposed methods are explained in Sections 3, 4 and 5. Simulation results are provided in Section-6 and conclusions are drawn in Section-7.

II. BACKGROUND INFORMATION

A. *Used Notation and SC Decoding*

In our manuscript, $u_1^N = (u_1, u_2, \dots, u_N)$ is used to represent N-bit information vector. After polar encoding operation, the code-bit vector $x_1^N = (x_1, x_2, \dots, x_N)$ is obtained and it is transmitted through communication channel after digital modulation. At the output of the communication channel the vector $y_1^N = (y_1, y_2, \dots, y_N)$ is observed.
At the destination side, using the received word $y_1^N = (y_1, y_2, \dots, y_N)$, information bits are estimated successively using the LRs of the information bits. It is shown in [1] that the LRs of the information bits can be calculated in a recursive manner. The decision is made according to.

$$\hat{u}_i = \begin{cases} 0 & \text{if } LR(\hat{u}_i) \geq 1 \\ 1 & \text{otherwise} \end{cases} \quad (1)$$

where

$$LR(\hat{u}_i) = \frac{Prob(\hat{u}_i = 0|y)}{Prob(\hat{u}_i = 1|y)}.$$

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B. AWGN Channel

Let $u_i, x_i \in \{0,1\}$ $i = 1, \dots, N$ be the information and polar code bits. The output of the discrete additive white Gaussian noise (AWGN) channel can be written as

$$r_i = y_i + n_o$$

where y_i are the symbols obtained by digitally modulating polar code bits x_i, and n_o is a random variable with zero mean and variance σ^2. Using the received word $r_1^N = (r_1, r_2, \dots, r_N)$ at the destination side, information bits u_i are estimated successively. For BPSK modulated code-bits, i.e., $y_i = 2 \times x_i - 1$ and $y_i \in \{-1, 1\}$, the conditional probability density functions, i.e., maximum likelihood probabilities, at the receiver side can be calculated as

$$p(r_i|y_i = \pm 1) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(r_i \mp 1)^2}{2\sigma^2}\right]$$

For BPSK modulation, likelihood ratio (LR) for y_i is defined as

$$LR(y_i) = \frac{p(r_i|y_i = -1)}{p(r_i|y_i = 1)} \qquad (2)$$

whose logarithmic form, i.e., log-likelihood ratio (LLR), is calculated as

$$LLR(r_i) = \ln\left(\frac{p(r_i|y_i = -1)}{p(r_i|y_i = 1)}\right) \to LLR(r_i) = \frac{2r}{\sigma^2}. \qquad (3)$$

C. Rayleigh Fading Channel

The output of the discrete Rayleigh fading is expressed as

$$r_i = h_i y_i + n_o$$

where h_i is a random variable with Rayleigh distribution, y_i and n_o are the same signals as defined in the previous sub-section. Maximum likelihood probability for y_i can be calculated using

$$p(r_i|y_i, h_i) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(r_i - h_i y_i)^2}{2\sigma^2}\right] \qquad (4)$$

Assuming that Rayleigh channel coefficients h_i are perfectly known at the receiver and BPSK modulation is used, log-likelihood ratio (LLR) for y_i can be calculated as

$$LLR(r_i) = \ln\left(\frac{p(r_i|h_i, y_i = -1)}{p(r_i|h_i, y_i = 1)}\right) \to LLR(r_i) = \frac{2hr}{\sigma^2} \qquad (5)$$

III. VIRTUAL ERASURE CHANNEL

Information bits are polar encoded and transmitted through continuous channel, such as AWGN or Rayleigh, after digital modulation. At the receiver side, before starting to the decoding operation, we consider a virtual erasure channel (VEC) and pass the received signal through virtual erasure channel as illustrated in Fig. 1 where the virtual erasure channel differs from the BEC in the channel inputs and outputs. VEC accepts real numbers as inputs from the continuous channel (AWGN/Rayleigh) and outputs either erasures or real values (from AWGN/Rayleigh channel). The operation of virtual erasure channel is described in

$$\tilde{r}_l = \begin{cases} 0, & \text{if } -\mu_t \le r_i \le +\mu_t \\ r_i, & otherwise \end{cases}$$

where μ_t is the threshold value, r_i is the output of the AWGN and $\tilde{r}_l$ is the output of VEC. The virtual erasure channel in Fig. 1 can also be considered as one-step non linear quantizer. Likelihood and log-likelihood values at the output of the VEC for BPSK modulation can be calculated as

$$LR(\tilde{r}_l) = \frac{P(\tilde{r}_l|y_i = -1)}{P(\tilde{r}_l|y_i = 1)} \to LLR(\tilde{r}_l) = \ln\frac{P(\tilde{r}_l|y_i = -1)}{P(\tilde{r}_l|y_i = 1)} \qquad (6)$$

where $LR(\tilde{r}_l)$ can be greater than, smaller than or equal to 1, and $LLR(\tilde{r}_l)$ can be greater than, smaller than or equal to 0.

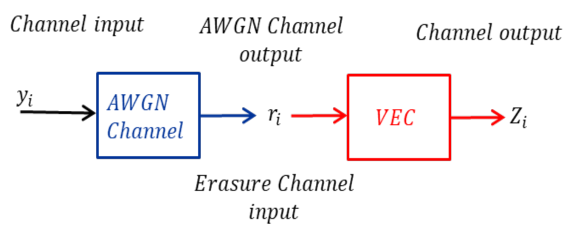

Fig.1. Channel model for AWGN+EC

IV. THRESHOLD DETERMINATION

For the determination of threshold, we consider two approaches. In the first approach, threshold value is calculated using the likelihood-ratio of the transmitted symbol computed at the receiver side [7]. In the second method, a predefined threshold is used for LLR values according to the soft information received through the channel.

A. Threshold μ_t determination for AWGN channel

We assume that data bits u_i are encoded, and the obtained polar code bits x_i are BPSK modulated resulting in y_i which are transmitted over AWGN channel. Frame length is N and r_i is the received symbol. The conditional probability density function $p(r_i|y_i)$ given by

$$p(r_i|y_i) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(r_i - y_i)^2}{2\sigma^2}\right] \qquad (7)$$

The graphs of $p(r_i|y_i = -1)$ and $p(r_i|y_i = 1)$ are depicted in Fig. 2. We define the absolute difference function $\delta(y)$ between the two conditional probability density functions as

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$$\delta(r) = |p(r|y=1) - p(r|y=-1)| \quad (8)$$

The maximum value of $\delta(r)$, i.e., δ_{max}, can be determined taking the derivative of $\delta(r)$ and equating it to zero as in

$$\delta(r) = \frac{1}{\sqrt{2\pi\sigma^2}}\left[e^{\frac{-(r-1)^2}{2\sigma^2}} - e^{\frac{-(r+1)^2}{2\sigma^2}}\right] \rightarrow \quad (9)$$

$$\left.\frac{d\delta(r)}{dr}\right|_{r=\mu_m} = 0 \rightarrow$$

$$\frac{1}{\sqrt{2\pi\sigma^2}}\left[\frac{1-r}{\sigma^2}e^{-(r-1)^2/2\sigma^2} - \frac{-(1+r)}{\sigma^2}e^{-(r+1)^2/2\sigma^2}\right] = 0 \quad (10)$$

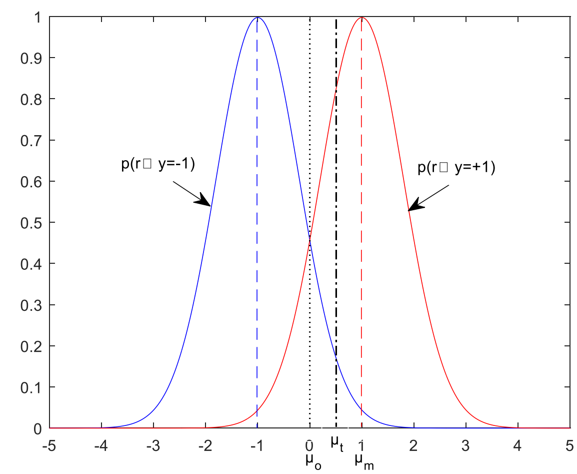

Fig. 2 threshold μ_t proposed over AWGN channel.

From (10), we obtain

$$(1-\mu_m)e^{\frac{\mu_m}{\sigma^2}} = -(1+\mu_m)e^{\frac{-\mu_m}{\sigma^2}} \quad (11)$$

which can be solved numerically by using Newton Raphson method [8] and for various values of $\sigma^2(0.1 \rightarrow 0.9)$. The value of r at which $\delta(r)$ is maximum is found as $\mu_m \approx 1.04$ which is almost equal to the mean value of $p(r|y=1)$. For the VEC we can choose the threshold level as $\mu_t = \mu_m/4$. In Table I, *LR* values at the output of VEC for threshold interval $[-\mu_t \ \ \mu_t] \rightarrow [-0.25 \ 0.25]$ are shown.

TABLE I

LR values at the output of VEC for $\sigma = 0.631$, R=0.5 and $\mu_t = \pm 0.25$

Input Symbols	Encoded Symbols	AWGN Channel	AWGN +VEC	LR
1	1	0.6760	0.6760	0.0335
1	0	-0.5134	-0.5134	13.189
1	0	0.7364	0.7364	0.0247
1	1	0.4768	0.4768	0.0911
0	1	0.7865	0.7865	0.0192
1	0	0.1118	0	1
0	0	-0.0991	0	1
1	1	1.2775	1.2775	0.0016

B. Threshold use for likelihood ratios

The threshold level calculated from the received data in the previous section depends on σ, i.e., it depends on SNR. The threshold levels can also be determined considering likelihood ratios in a deterministic manner. In this paper, we consider three threshold intervals $[-\mu_t \ \ \mu_t] \rightarrow [-0.5 \ 0.5]$, $[-\mu_t \ \ \mu_t] \rightarrow [-1 \ \ 1]$ and $[-\mu_t \ \ \mu_t] \rightarrow [-2 \ 2]$, the symbol that passes through VEC having *LLR* value falling into the range $[-\mu_t \ \ \mu_t]$ is erased. The output of the VEC is calculated as

$$LLR_{VEC}(\tilde{r}_i) = VEC(LLR(r_i)) \rightarrow \begin{cases} 0 & -\mu_t < LLR(r_i) < \mu_t \\ LLR(r_i) & \text{else} \end{cases} \quad (12)$$

V. Improved Polar decoders employing VECs

VECs can be used to improve the performance of SC decoders. In this section, we propose an improved polar decoder structure utilizing two VECs.

A. Improved SC Decoder with Two VECs

To alleviate the information loss and improve the decision robustness, the output of the AWGN channel of Fig.1 passes through two parallel VEC with threshold levels $|\mu_{t1}|$=1 and $|\mu_{t2}|$=2. The outputs of the VECs are calculated as

$$\text{VEC}_1:\ LLR(\tilde{r}_1) = \begin{cases} 0 \ \text{ if } -1 \le LLR(\tilde{r}_1) \le 1 \\ LLR(\tilde{r}_1) \ \text{ otherwise} \end{cases} \quad (13)$$

$$\text{VEC}_2:\ LLR(\tilde{r}_2) = \begin{cases} 0 \ \text{ if } -2 \le LLR(\tilde{r}_2) \le 2 \\ LLR(\tilde{r}_2) \ \text{ otherwise} \end{cases} \quad (14)$$

The outputs of the VECs are fed to the decoder unit as depicted in Fig. 3 where we utilize two SC decoders, one of decoders processes the output of VEC1 whereas the other processes the output of VEC2. Once the decoding operation for all information bits is complete, sum of the absolute log-likelihood ratios, i.e.. *LLRs*, is performed as

$$S_{PD_1} = D1{:}\sum_i |LLR_i| \quad S_{PD_2} = D2{:}\sum_i |LLR_i| \quad (15)$$

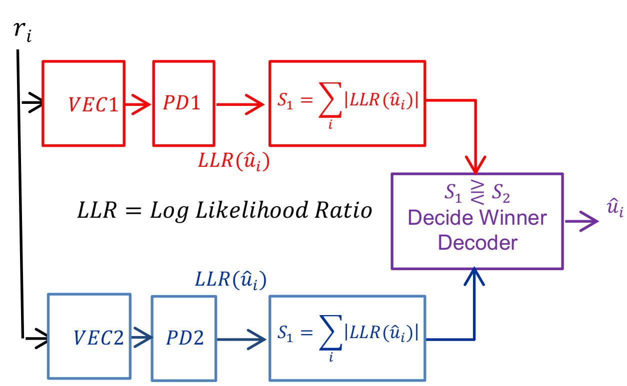

Fig.3. Block diagram for 2- parallel SC decoding operation

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where LLR_i indicates the log-likelihood ratio for information bit u_i for the corresponding decoders. The winner decoder is chosen considering

$$PD_w = \begin{cases} PD_1 & \text{if } S_{PD1} \geq S_{PD2} \\ PD_2 & \text{otherwise.} \end{cases} \quad (16)$$

The estimated bits of the winner decoder are accepted as the result of the decoding operation. Absolute LLR terms in (15) can be considered as information amount carried by LR and summing all the $|LLR_i|$ values, we get the total information content of the decoder under concern. It is similar to the concept of entropy in information theory with some differences.

VI. Simulation Results

Simulations are performed for frame lengths 128, 256 and bit-error-rate (BER) and frame error rate (FER) performance curves are obtained. BPSK modulation is used. For the first type of simulation, only a single VEC is used and the performance curves are obtained for two different threshold values. The code rate is R=0.5. In Figs. 4-7, performance curves are obtained for AWGN+VEC channel for different frame lengths using the method proposed in section IV-A.

In Figs. 8-11, simulations results are presented for AWGN and Rayleigh channels for LLR based threshold approach proposed in sections IV-B,V-A. In this second type of simulation, two VECs are used in parallel, and threshold levels used by these VECs are $\mu_t(LLR: D_1)$=±1 and $\mu_t(LLR: D_2)$=±2. At the receiver side, two decoders shown in Fig. 3 are employed for decoding operation. In performance Figures, the lines with caption markers '**' indicate that the simulation results are obtained assuming that perfect channel knowledge is available at the receiver side, and if the sequence associating with one of the decoded paths matches the transmitted frame, it is accepted as the decoder result, otherwise, the path having the largest absolute LLR sum is chosen. The results show that a significant polar decoding performance improvement is achieved for any frame length.

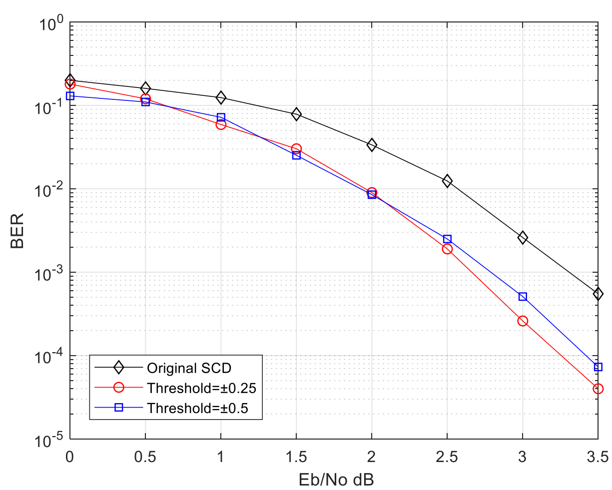

Fig 4. BER performance under different threshold levels for frame- length N=128, and rate R=0.5 over AWGN Channel.

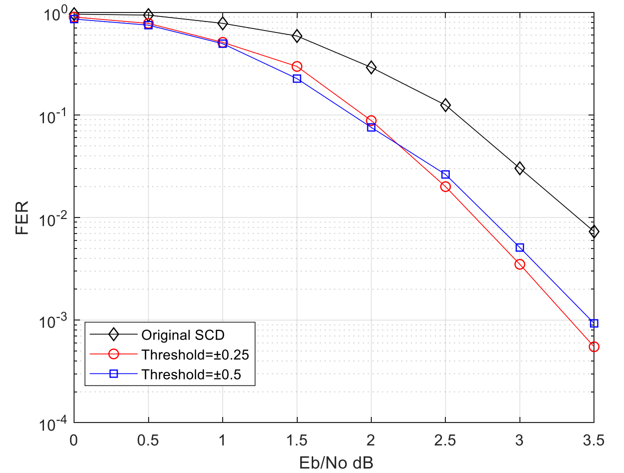

Fig 5. FER performance under different threshold levels for frame- length N=128, and rate R=0.5 over AWGN Channel.

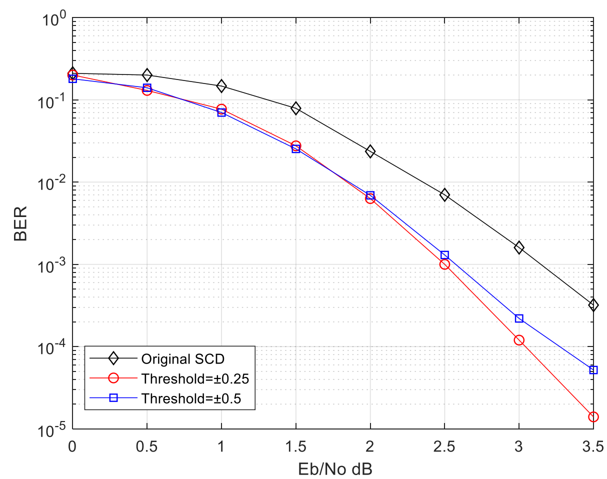

Fig 6. BER performance under different threshold levels for frame- length N=256, and rate R=0.5 over AWGN Channel.

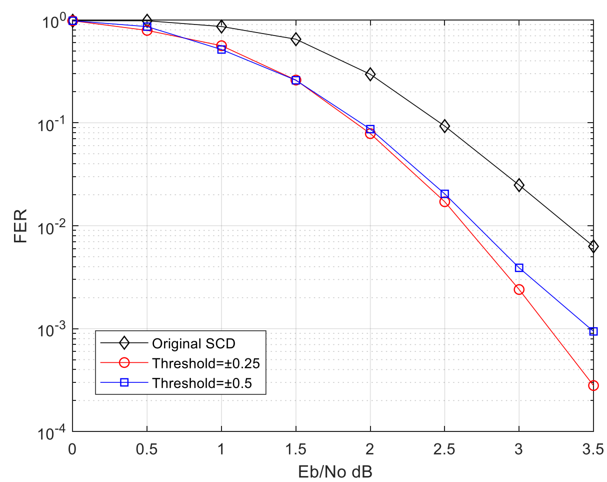

Fig 7. FER performance under different threshold levels for frame- length N=256, and rate R=0.5 over AWGN Channel.

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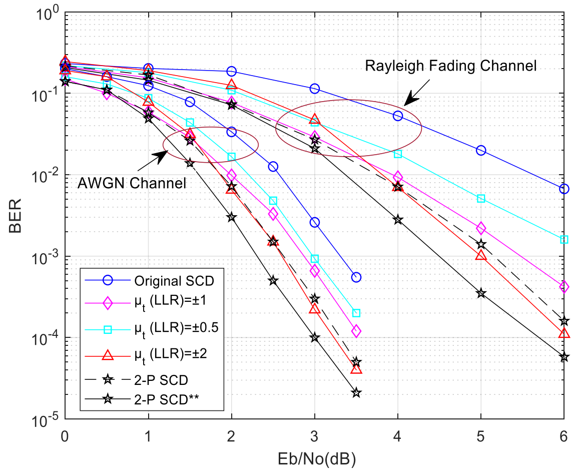

Fig 8. BER performance under different threshold levels for frame- length N=128, and rate R=0.5 over AWGN and Rayleigh Channel.

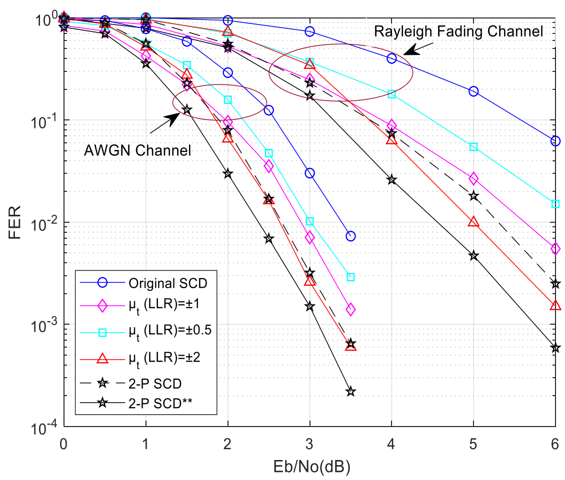

Fig 9. FER performance under different threshold levels for frame- length N=128, and rate R=0.5 over AWGN and Rayleigh Channel.

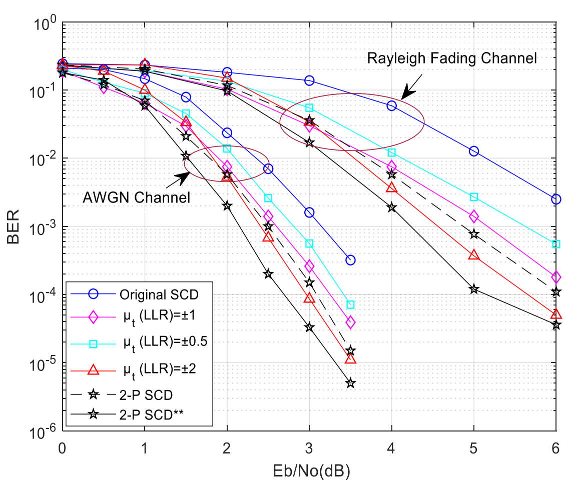

Fig 10. BER performance under different threshold levels for frame- length N=256, and rate R=0.5 over AWGN and Rayleigh Channel.

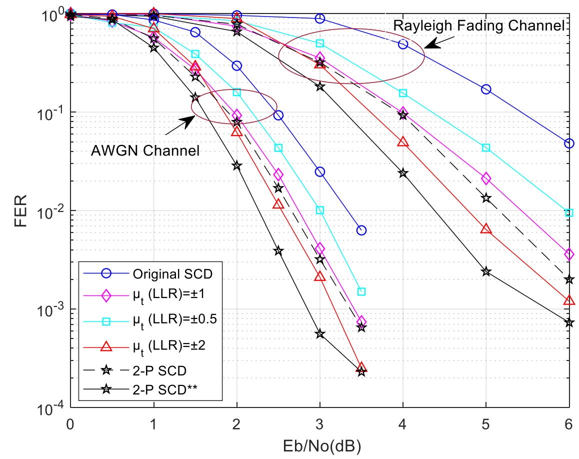

Fig 11. FER performance under different threshold levels for frame- length N=256, and rate R=0.5 over AWGN and Rayleigh Channel.

I. CONCLUSION

In this paper, we proposed the use of virtual erasure channels at the receiver side to boost the performance of polar decoders. The symbols passing through VECs are erased if they fall into a predetermined threshold interval. For BPSK modulated transmission schemes, we proposed techniques for the determination and use of threshold intervals for VECs. It is also proposed that more than single VECs can be employed in parallel and for the receiver part of such systems more than one decoder can run in parallel. And, we proposed the use of absolute *LLR* summation, which can be considered as the total information content of the decoder, to decide on the winner decoder. Simulations are performed for AWGN and Rayleigh channels, and it is seen that the performance of SC polar decoders is significantly improved such that 1dB improvement is achieved for AWGN channels and 2dB improvement is obtained for Rayleigh fading channels.

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