Calculation of the values of the constants of Pelofsky’s equation and comparison with their practical values for certain polymer solutions

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***Abstract*— This study investigates the applicability of Pelofsky's equation to nine polymer aqueous solutions, including Carboxymethyl cellulose, Polyanionic cellulose, Polyacrylamide, Polyvinylalcohol, Polyvinylpyrrolidone, and polyethylene glycol with varying molecular weights. The viscosity and surface tension of these solutions were experimentally determined. The constants A and B of Pelofsky's equation were calculated, but the predicted values did not match the experimental results obtained from the slope of the Ln(ɣ) graph. This discrepancy is attributed to the empirical nature of the equation. Future research can explore the applicability of Pelofsky's equation to a broader range of polymer solutions and examine its compatibility with the relationship between viscosity and surface tension**

Keywords— ***surface tension, viscosity, Pelofsky's equation, polymer solutions, correlations..***

I. Introduction

The physical properties of polymers depend on their molecular weight and physical structures. [1] Polymers are distinguished by the arrangement of the repeating units. A polymer formed from a single monomer is a homogeneous polymer, and if it contains several monomers it is a copolymer. These include random, alternating, block and graft polymers. [1] The viscosity of polymers play an important role in the production, processing and application of it. [2] Polymers generally have high viscosity.[2] The viscosity of the polymer solution depends on the length scale of flow. When the length scales smaller than the polymer coil size, the viscosity is close to that of the solvent. [3] The viscosity of polymer solutions is important for polymer synthesis and recycling. Polymerization reactions can be hampered by diffusional limitations once a viscosity threshold is reached, and viscous solutions complicate clean-up steps during the dissolution-precipitation technique. [4] Surface tension is a characteristic property of liquids. This phenomenon is due to the force of attraction between the molecules on the surface. [5] The surface tension of solutions is an important physical and chemical property of surfactant molecules, different methods are used to determine it depending on the time scale of surface tension decay appropriate for the particular application. [6] Viscosity is the measure of friction in the fluid, and closely related to surface tension, but surface tension is the tendency of the fluid to stretch due to the attractive forces between molecules. [7] The relationship between surface tension and viscosity is conceptually important and can be used to test the validity of measured data. [8] Chideme et al. The effect of fluid properties - viscosity, surface tension - and air pressure on the mean Sauter diameter of evaporating nozzles using a dual liquid evaporator was studied, and equations were derived that quantify these effects across a range of fluid behaviors, with emphasis on viscous and non-viscous fluids. [9] Mahdiye et al. demonstrate that a classic method for measuring the surface tension of liquid droplets, based on the analysis of the shape of a sessile droplet, can be effectively scaled down to measure the interfacial tension between a macromolecule-rich droplet phase and its co-existing macromolecule-poor continuous phase. The connection between droplet shape and surface tension relies on the density difference between the droplet and its surroundings. This can be determined with small sample volumes in the same setup by measuring the droplet sedimentation velocity. An interactive mat lab script for extracting the capillary length from a droplet image is included in the ESI. [10] Shao et al. A method for obtaining the elasticity, surface tension, and viscosity of ultrasonically levitated gel drops is presented. A method is described for obtaining elasticity, surface tension, and viscosity, and the method is experimentally demonstrated for surface tension and viscosity. [11]

A. The theory

This study relies on several theoretical concepts, including:

1) Surface tension: 1. The surface tension (ɣ) of a solution is related to the mass (m) of a drop through the equation(1) : [12]

$$m\,g = 2\pi\, r\, \gamma \qquad (1)$$

where g is the acceleration due to gravity (9.8 m/s²) and r is the radius of the wetted tip.

2) Viscosity: Viscosity: The viscosity of a solution is related to its flow time through the Poiseuille equation: (2): [13]

$$\eta = \frac{\pi\, \Delta P\, r^{4}\, t}{8\, V\, L} \qquad (2)$$

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where η is the coefficient of viscosity, t is the flow time, V is the volume of liquid, ΔP is the pressure change, and L is the length of the viscometer.

3)Viscosity-surface tension correlations: Pelofsky's equation proposes a relationship between viscosity (Ƞ) and surface tension (ɣ), which is shown in the following equation(3): [14, 15]

$$Ln\, \gamma = Ln\, A + \frac{B}{\eta} \qquad (3)$$

Where A and *B* are substance-dependent constants, [15] that can be determined through fitting coefficients Ln A and B. [14]

B. Materials and Methods:

1) Materials: The study used various polymers, including Carboxymethyl cellulose (CMCHV, CMCLV) and Polyanionic cellulose (PAC LV) provided by National Corporation Jowfe Oil Technology, and Polyacrylamide (PMA), Polyvinylalcohol (PVA), Polyvinylpyrrolidone (PVP), and polyethylene glycol (PEG) with different molecular weights (Mw 8000, Mw 10000, Mw 12000) supplied by Sigma-Aldrich GMBH.

2) Samples preparation: Two solutions of each polymer with different concentrations (19%, 31%, 43%, 55%) were prepared by dissolving them in distilled water with stirring at room temperature (24°C)

3) Measurements & calculations

a) Surface tension: The surface tension was calculated using the droplet shape method and equation (1), which relates the mass of the droplet to the surface tension. [12]

b) Viscosity: The viscosity was calculated from the flow time measured using a glass viscometer and equation (2).13]

c) Viscosity-surface tension correlations: The constants A and B were calculated using equation (3), which represents the empirical relationship between viscosity and surface tension proposed by Pelofsky. [16]

4) Results and discussion:
The results are presented in Tables (1) and (2), which show the calculated values of surface tension and viscosity for the different polymer solutions. The PEG samples were separated in Table (2) to examine the effect of molecular weight on the constants A and B. The results indicate that the change in molecular weight of PEG does not have a clear relationship with the change in the constants A and B.

Table.1 Values Measurements and calculations from paragraph (3

Polymer	C %	ɣ	η	A	B
CMCHV	19	0.3352	151.3851		
	31	0.4369	191.894	0.131434	-141.7294
	43	0.6866	353.0556		
	55	0.7362	948.4181		
CMCLV	19	0.3543	102.6536	0.158904	-82.31158
	31	0.38815	116.4905		
	43	0.4618	153.3044		
	55	0.6274	357.2573		
PACLV	19	0.1745	65.4085	0.013028	-81.56331
	31	0.2466	96.4539		
	43	0.3469	181.3672		
	55	0.39324	268.0771		
PVA	19	0.1493	70.50695	0.143833	-2.62974
	31	0.1529	187.1298		
	43	0.1534	251.2232		
	55	0.15402	426.3841		
PVP	19	0.04531	8.8004	0.012299	-11.47598
	31	0.0905	18.7452		
	43	0.09641	20.9054		
	55	0.11631	31.7631		
PAM	19	0.14624	8.6471	0.05696	-8.1538
	31	0.1841	10.2541		
	43	0.2714	23.01889		
	55	0.3371	75.6228		

Table. 2 Values Measurements and calculations from paragraph (3) for PEG`s

MW	C %	ɣ	η	A	B
8,000	19	0.13256	3.4461	0.02991	-5.1309
	31	0.15292	3.8		
	43	0.1841	4.4056		
	55	0.2023	4.8124		
10,000	19	0.09532	2.97978	0.03960	-2.61719
	31	0.1217	4.1564		
	43	0.1529	6.5194		
	55	0.1626	7.6024		
12,000	19	0.1246	2.7633	0.004418	-9.22756
	31	0.1841	3.1291		
	43	0.2466	3.4734		
	55	0.2864	3.68065		

a) Surface tension: The values of the surface tension coefficient given in Tables (1) and (2) were calculated for all samples using equation (1) after obtaining the weight of the solution drops using the drop method.

b) Viscosity: Using equation (2), the viscosity coefficient was calculated as a function of the flow time measured in the experiment and the results were recorded in tables (1) and (2).

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c) Viscosity-surface tension correlations: From equation (3), the values of A and B were calculated for both samples and the results were recorded in tables (1) and (2).The equation (3) is a relationship between surface tension and viscosity. It can be applied to purely organic and inorganic fluids and mixtures.[16] Belowski and Schonhorn suggested that A which is the intercept of the curve in the relationship between the inverse of the viscosity coefficient and the logarithm of the surface tension coefficient [17] could be an indicator of the surface tension of the liquid at the temperature where $\eta = \infty$They attempted to find a correlation between the temperature where $\eta \rightarrow \infty$ and the homogeneous crystallization temperature.[8] When plotting $\frac{1}{\eta}$ versus Ln(ɣ), the slope of the line indicating the inverse relationship is the value of the constant B.[8,18] The relationship between the inverse of the viscosity coefficient and the logarithm of the surface tension coefficient for all samples is shown in figure (1) and from there the slopes of the lines in the figure represents the experimental values of the constant B.

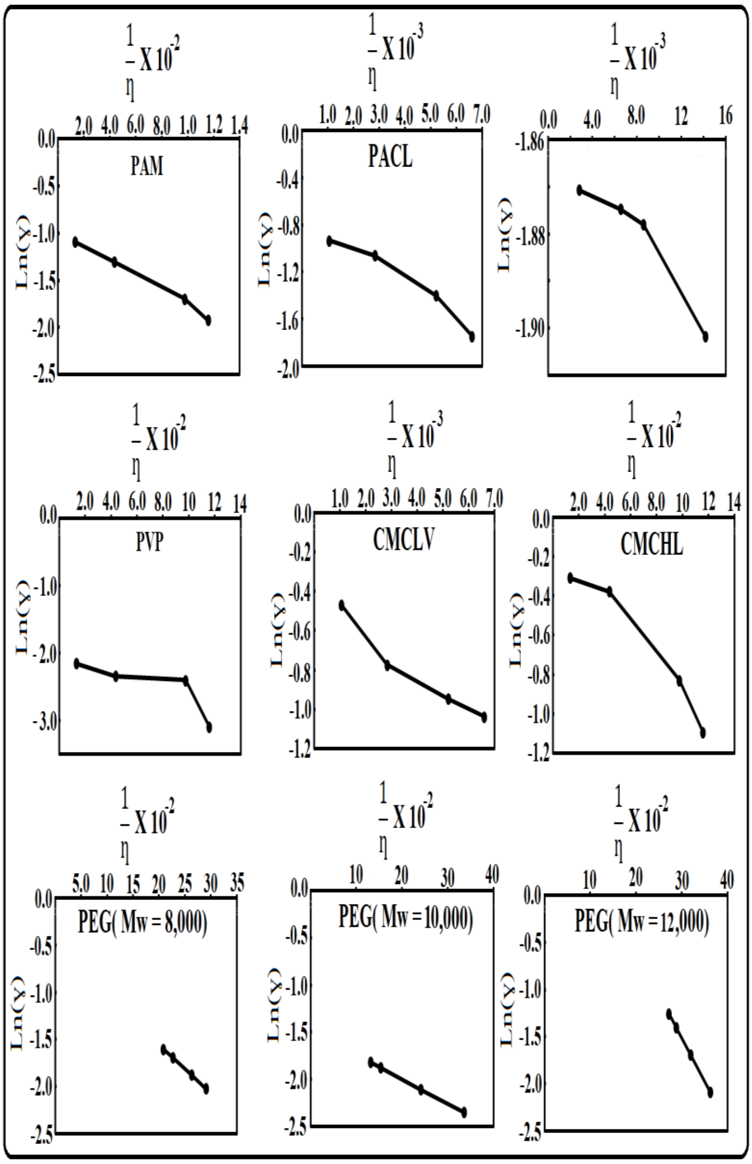

Fig. 1 The relationship between the coefficient of the viscosity coefficient and the logarithm of the surface tension coefficient.

Table.3 The constant B is equal the slope of the lines in the figure (1).

Polymers	A	B
CMCHV	-2.5	-7.69942
CMCLV	-0.39	-99.2178
PACLV	-0.7	-144.447
PVA	-1.865	-7.89175
PVP	-2.1	-7.18629
PAM	-1.0	-7.89175
PEG(Mw= 8,000)	-0.49	-5.1348
PEG(Mw= 10,000)	-1.4	-2.61111
PEG(Mw= 12,000)	0.00	-9.22742

The calculated values of constants A and B using Equation (3) in Tables (1) and (2) were compared with the values obtained from Figure (1) in Table (3). For constant B, significant differences were observed for CMCHV, CMCLV, and PACLV, while the values were close for PEG with molecular weights of 8000, 10000, and 12000. However, large discrepancies were found for constant A in all cases. These differences may be attributed to the empirical nature of Equation (3). [19, 20].

Conclusions

In conclusion, this study measured the surface tension and viscosity of aqueous polymer solutions to calculate the constants A and B of Pelofsky's equation, which relates surface tension and viscosity. However, the calculated values did not match those obtained from the slope of the Ln(ɣ) vs. plot, likely due to the empirical nature of the equation. While Pelofsky's equation may not provide a perfect fit, it can still be applied to various polymer solutions to explore its compatibility with the relationship between viscosity and surface tension. Further research can investigate the equation's applicability and potentially provide physical meaning to the constants, particularly the slope B, which Pelofsky correlated with molecular weight for certain polymer

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